# CH-5

## Permutation & Combination

Permutation — Order imp. hai

Combination — Order imp. nhi hai

- Fundamental Principle of Counting
- Permutation (Arrangement)
- Combination (Selection)

# Understanding Factorial

n factorial → Product of 1st n natural numbers
$n!$ or $\lfloor n$

$\boxed{\text{example} \Rightarrow 4! = 4 \times 3 \times 2 \times 1 = 24}$

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$

$6! = 720$
$7! = 5040$
$8! = 40320$

$$7! = 7 \times 6!$$
$$= 7 \times 6 \times 5!$$

$$n! = n \times (n-1)!$$
$$= n \times (n-1) \times (n-2)!$$

Negative No.s & fractions ka kbhi bhi factorial nhi niklega

## Some basic thing

→ $10! = ?$ $\quad 8!$

$10! = 40320 \times 9 \times 10$

$= 3628800$

→ $\dfrac{15!}{12!} = \dfrac{15 \times 14 \times 13 \times \cancel{12!}}{\cancel{12!}}$

$= 2,730$

## Fundamental Principle of Counting (FPC)

### Addition Rule

$H_1$ $\quad$ $B_1$

$H_2$ $\quad$ $B_2$

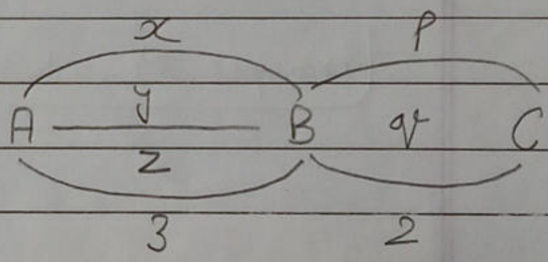
$H_3$

Honda (or) Bajaj

$3 + 2 = 5$

OR

### Multiplication Rule




$xp, xq, yp, yq, zp, zq$

$3 \times 2 = 6$

AB (and) BC

Yeh bhi karna hai & Yeh bhi karna hai

And

Example- 9 teams participated in competition. There are 3 prizes. In how many ways these prizes can be distributed.

| | |
|---|---|
| $1^{st}$ | 9 |
| $2^{nd}$ | 8 |
| $3^{rd}$ | 7 |

$$9 \times 8 \times 7 = 504$$

# Permutation (Arrangement) order important hai, matter karta hai

→ Number of way of arranging r things out of n

is $^nP_r = \frac{n!}{(n-r)!}$ , where $n \geq r$

[Positive Integer]

n = choice
r = arrange

6 seats
3 Friends

$^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!}$

$= \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 120$

→ Number of ways of arranging n distinct things amongst themselves is $= n!$

$^9P_4 = 9 \times 8 \times 7 \times 6 = 3024$

$^{11}P_5 = 11 \times 10 \times 9 \times 8 \times 7 = 55440$

$^nP_0 = 1$ $[0! = 1]$

$^nP_n = n!$

Conse. No.

TRICK

$^nP_2 = 72 = 8 \times 9$

$n = 9$

$^7P_0 = 1$

$^5P_1 = 5$

$^4P_4 = 24$

Example- SOHAN → 5! = 120

SSOOOHHAAAN → $\frac{11!}{2!\,3!\,2!\,3!}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times \cancel{6} \times 5 \times \cancel{4} \times \cancel{3!}}{1 \cancel{2} \times 1 \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{3!}}$

$= 2,77,200$

Case 3

→ Number of ways of arranging n things out of which p are similar, another q are similar, another r are similar ...... $= \frac{n!}{p!\,q!\,r!\ldots}$

Example- (i) COMPETITION → $\frac{11!}{2! \times 2! \times 2!}$

$= \frac{11!}{(2!)^3} = \frac{11!}{8}$

$= 4,989,600$

(ii) APPETITE $= \frac{8!}{(2!)^3} = \frac{8!}{8} = 5040$

(iii) Ratio of CALCUTTA $= \frac{8!}{2!\,2!\,2!} = 5040$

AMERICA $= \frac{7!}{2!} = 2520$

$\frac{2}{1}$

$\boxed{2:1}$

OR

$$\frac{CALCUTTA}{AMERICA} = \frac{8!}{2!2!\cancel{2!}} \times \frac{\cancel{2!}}{7!} = \frac{\overset{2}{\cancel{8}} \times \cancel{7!}}{{}_1\cancel{4} \times \cancel{7!}}$$

$$= \frac{2}{1} \text{ or } 2:1$$

Case 1

## ★ Permutation with Restrictions

Example- A C C E P T E D

(i) Such that vowels are always together.

$$\underline{(A\,E\,E)}\ \underline{C}\ \underline{C}\ \underline{P}\ \underline{T}\ \underline{D} \rightarrow \frac{6!}{2!} \times \frac{3!}{2!}$$

$$= 1080$$

(ii) Such that all consonants are always together

$$\underline{(C\,C\,P\,T\,D)}\ \underline{A}\ \underline{E}\ \underline{E} \rightarrow \frac{4!}{2!} \times \frac{5!}{2!}$$

$$= 720$$

Q. There are 5 boys & 3 girls, in how many ways they can be arrange in a row such that all girls are always together.

i) $\underline{(G_1 G_2 G_3)}\ \underline{B_1}\ \underline{B_2}\ \underline{B_3}\ \underline{B_4}\ \underline{B_5} \rightarrow 6! \times 3!$

$= 4320$

ii) $\underline{(B_1\,B_2\,B_3\,B_4\,B_5)}\ \underline{G_1}\ \underline{G_2}\ \underline{G_3} \rightarrow 4! \times 5!$

$= 2880$

Note : Jinko sath me nahi rakhna unko hamesa baad me jamate hai

Q. 5 boys 3 girls
In how many ways they can be arranged so that no two girls are together.

× B × B × B × B × B ×

$5! \times 6 \times 5 \times 4 = 14,400$

Q. 6 friends A, B, C, D, E, F stand for a photograph such that.
A & B will have always on person b/w them

___ ___ ___ ___ ___ ___

$A_1$ $B_3$

$A_2$ $B_4$

$A_3$ $B_5$

$A_4$ $B_6$

$4 \times 2 \times 4! = 192$

SUPER SHORTCUT
If 2 & only 2 items are never together
$(n-2)(n-1)!$
or
Total Arrangement − Always Together

→ सब पर लगेगी (General Approach)

Step 01 Jinke natak na ho unko arrange karo

Step 02 Unke sath - paas wali jagah par kki Never together waale arrange kar do /

Q. The sum of all 3 digit number containing the digits 3, 1, 7 without reptitions is

317
371
713
731
173
137
2442

Sum of digits × 111 ... n times × $(n-1)!$

11 × 111 × 2!

2442

TRICK: Sum of given digits × 111 ... n times × $(n-1)!$

No. of Factors $N = a^p \times b^q \times c^r$ ...

$(1+p)(1+q)(1+r) \ldots - 1$

Case 4 **Circular Permutations**

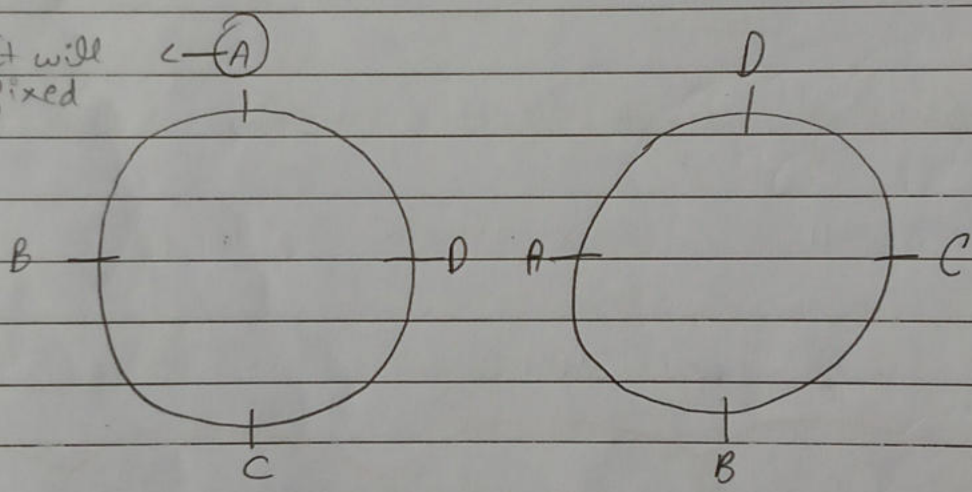
In a circle n distinct things can be arranged among themselves in $(n-1)!$ ways.

A , B , C , D

___ ___ ___ ___

$4! = 24$

It will be fixed




$3! = 6$

Phele ek ki jagah fix karo phir formula lagao.

Case 2 Permutations when objects can be repeated

Formula → Repetition

$$\boxed{n^r}$$

[ Chupki → Repetition is not allowed

Case 5 Permutations in case of Division & Distribution

Formula →

$$\frac{\text{Total items}!}{\underset{n_1!}{\text{group}.1}\ \underset{n_2!}{\text{group}.2}\ m!} \times \text{Person}!$$

Distribution

m = no. of same groups

## Necklace Formula

n jewels can be arranged to form a necklace in $\frac{(n-1)!}{2}$ ways.

# Combination (Selection) [Order does not matter]

→ Number of way of selecting r things out of n in

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}, \text{ where } n \geq r$$

[Commity bnane wale or team bnane wale questions]

★ Difference b/w Perm. & Comb.

**Perm.**

A B C

$^{3}P_{3} = 3! = 6$

ABC
ACB
BAC
BCA
CAB
CBA

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3!}{0!}$

$= 3! = 6$

order matters

**Comb.**

A B C

$^{3}C_{3} = 1$

Team

A B C
B A C

members in both are same

Here, order doesn't matters

$^{3}C_{3} = \frac{3!}{3!(3-3)!} = \frac{\cancel{3!}}{\cancel{3!} \cdot 0!} = 1$

Ex- $^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2} = 120$

$^{11}C_5 = \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} = 462$

$^nC_0 = 1$
$^nC_1 = n$
$^nC_n = 1$
$^nC_r = {^nC_{n-r}}$

- $^nC_{r_1} = {^nC_{r_2}} \rightarrow r_1 + r_2 = n$
- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

Cons. No.
TRICK
$^nC_2 = 15$ first double the no.
$= 30 = 6 \times 5$
$n = 6$

## Important Results

1. $^nP_r = r! \times {^nC_r}$ OR $\frac{^nP_r}{^nC_r} = r!$

Ex $n = 6$, $r = 3$

$^6P_3 = 3! \times {^6C_3}$

$6 \times 5 \times 4 = \cancel{3!} \times \frac{6 \times 5 \times 4}{\cancel{3!}}$

$120 = 120$

$\frac{^nP_6}{^nC_6} = r!$

$= 6! = 720$

ex- $^{15}P_5 = x \times {}^{15}C_5$

$x = 5!$

$x = 120$

ex- $^{n}P_r = 24 \times {}^{n}C_r$

$r = 4$

2. $^{n}C_r = {}^{n}C_{n-r}$

$^{n}C_a = {}^{n}C_b$ if $a + b = n$

ex. ☐ $^{7}C_3 = 35$ $\qquad$ $^{7}C_3 = {}^{7}C_4$ , Here $3+4 = 7$

$^{7}C_4 = 35$

☐ $^{8}C_3 = {}^{8}C_5$

☐ $^{11}C_4 = {}^{11}C_7$

☐ $^{15}C_x = {}^{15}C_9$ , $x = 6$

☆ The main use of it is:

☐ $^{25}C_{23} = {}^{25}C_2 = \frac{25 \times 24}{2} = 300$

☐ $^{17}C_{13} = {}^{17}C_4 = \frac{17 \times 16 \times 15 \times 14}{24} = 2,380$

☐ $^{18}C_x = {}^{18}C_{2x+3}$ , $x = ?$

$x + 2x + 3 = 18$

$3x + 3 = 18$

$3x = 15$

$\boxed{x = 5}$

3. Pascal's Rule

$${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$$

- $${}^{120}C_{50} = {}^{119}C_{50} + {}^{119}C_x$$

$x = r - 1$

$= 50 - 1 = 49$

- $${}^{x}C_5 = {}^{13}C_5 + {}^{13}C_4$$

$x = n + 1$

$= 13 + 1 = 14$

- $${}^{15}C_6 + 2\,{}^{15}C_5 + {}^{15}C_4 = ?$$

~~Same~~ sabse bda $\qquad {}^{15+2}C_6 = {}^{17}C_6$

6

- $${}^{20}C_8 + 2\,{}^{20}C_7 + {}^{20}C_6 = {}^{x}C_8$$

$x = 20 + 2 = 22$

- $${}^{n}C_{r+1} + 2\,{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+2}C_{r+1}$$

## Atleast One

$\Rightarrow \; {}^nC_0 + {}^nC_1 + {}^nC_2 \; \ldots\ldots\ldots + {}^nC_n = 2^n$

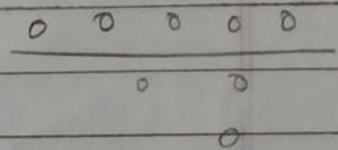
$\rightarrow \; {}^nC_1 + {}^nC_2 \; \ldots\ldots\ldots + {}^nC_n = 2^n - 1$

## Formula Related to Geometry

① n points

- → No. lines $\quad {}^nC_2$
- → No. Triangle $\quad {}^nC_3$
- → No. Quadrilaterals $\quad {}^nC_4$

② n points $\quad p \rightarrow$ Colinear



- → no. lines $\quad {}^nC_2 - {}^pC_2 + 1$
- → no. Triangles $\quad {}^nC_3 - {}^pC_3$

③ No. of diagonals of polygon $= {}^nC_2 - n$

④ m horizontal ∥ lines

n Verticle ∥ lines

No. of parallel Rectangle $= {}^nC_2 \times {}^mC_2$

## 3 Cases

(1) No. of handshakes
(2) No. of matches
(3) No. of tickets

$\left.\begin{array}{l} \text{No. of handshakes} \\ \text{No. of matches} \\ \text{No. of tickets} \end{array}\right\} {}^4C_2$

## Grouping Condition

(A) $10 \rightarrow 2, 2, 2, 4$

$$\frac{{}^{10}C_2 \times {}^8C_2 \times {}^6C_2 \times {}^4C_4}{3!}$$

$10 \rightarrow 2, 2, 3, 3$

$$\frac{{}^{10}C_2 \times {}^8C_2 \times {}^6C_3 \times {}^3C_3}{2!\,2!}$$

(B) Things are divided to different persons then we don't divide by factorial

10 Chocolates $\quad 2, 2, 3, 3$

$${}^{10}C_2 \times {}^8C_2 \times {}^6C_3 \times {}^3C_3$$

Ex 5 Boys 4 Girls (5 member Committee)

(A) Total → $^9C_5 = 126$

(B) It must include atleast 3 girls

| B | G |
|---|---|
| 2 | 3 |
| 1 | 4 |

$^5C_2 \times {}^4C_3 + {}^5C_1 \times {}^4C_4$

$10 \times 4 + 5 \times 1$

$= 45$

(C) Atmost 2 Boys

| B | G |
|---|---|
| 2 | 3 |
| 1 | 4 |

(D) It must include on particular Boy & one particular Girl.

$1 \times {}^7C_3 = 35$

(E) It must exclude one particular Boy.

$^8C_5 = 56$

eek Sir

# Special Cases of Combination

All or None | Selection of alike items

(i) All or None :

Q. Tunnu $\Rightarrow$ 6 friends
In how many ways can he invite ?

$$^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$$

$$1 + 6 + 15 + 20 + 15 + 6 + 1$$

$$= 64 \text{ ways}$$

Shortcut $\rightarrow$ $(\text{option})^n$ , $2^n = 2^6 = 64$

[1 or more] Atleast 1 $\rightarrow$ $(\text{option})^n - 1$ , $2^n - 1 = 2^6 - 1 = 64 - 1 = 63$

(ii) Selection of any no. of items if $n_1$ are alike, $n_2$ are alike , $n_3$ are alike ..... is $\rightarrow$

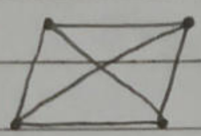
$$\underbrace{(n_1+1)\,(n_2+1)\,(n_3+1)}$$

atleast of 1 of each category $\rightarrow$ ans $-$ 1

(iii) Total Cases − Nhi chaiye = Chaiye

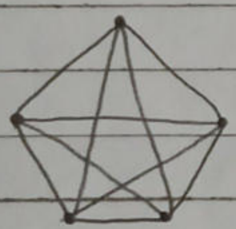
(iv) Combination in case of geometry figures

- No. of diagonals ? = ${}^{n}C_2 - n$



${}^{4}C_2 = 6$ lines

Jitne points hote hai utni sides bhi hoti hai



No. of diagonal $= {}^{n}C_2 - n$

$= {}^{5}C_2 - 5$

$= 5$

that lies on same line

- Collinear Points :- → All Points lies on same line

Q. 12 Points, 4 Collinear, No. of Triangles ?

${}^{12}C_3 - {}^{4}C_3$

$220 - 4 = 216$ triangles

- Parrellelogram